Phases of AdS black holes

jueves, 24 de noviembre de 2022 12:59

In AdS_{d+1}, with

$$I = \frac{1}{16\pi G}\int_{\mathcal{M}} d^{d+1}x\sqrt{-g}\left(R + \frac{d(d-1)}{L^2}\right) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^d x\sqrt{|h|}\,K$$

we have the solutions (now w/ $L=1$)

$$ds^2 = -\left(r^2 + k - \frac{\mu}{r^{d-2}}\right)dt^2 + \frac{dr^2}{r^2 + k - \frac{\mu}{r^{d-2}}} + r^2 d\Sigma^{(k)}_{d-1}$$

$k = \pm 1, 0$

$$d\Sigma^{(k)}_{d-1} = \begin{cases} d\Omega_{d-1} & k=+1 \\ d\vec{x}^2_{d-1} & k=0 \\ dH_{d-1} & k=-1 \end{cases}$$

For $\mu = 0$ there are all empty AdS_{d+1}

For $\mu \neq 0$ ($\mu > 0$ for $k = +1, 0$, $k=-1$ admits $\mu < 0$)

There is an outer event horizon at $r = r_+ > 0$

with $r_+^{d-2}(r_+^2 + k) = \mu$ (use r_+ as parameter)

The planar case can be obtained as the large-bh limit $r_+ \gg 1$ of the other two:

$r \to \lambda r$ $\qquad\qquad\qquad\qquad$ $\lambda \gg 1$

$r_+ \to \lambda r_+$ (or $\mu \to \lambda^d \mu$)

$t \to \frac{1}{\lambda} t$ (redshifting)

$d\Sigma_k \to d\vec{x}^2$ (flattening)

$d\Sigma_k \to dx^-$ (flattening)

Can compute temperature by going Euclidean $t \to -i\tau$

For $ds^2 = f(r)\, d\tau^2 + \frac{dr^2}{f(r)} + \ldots$

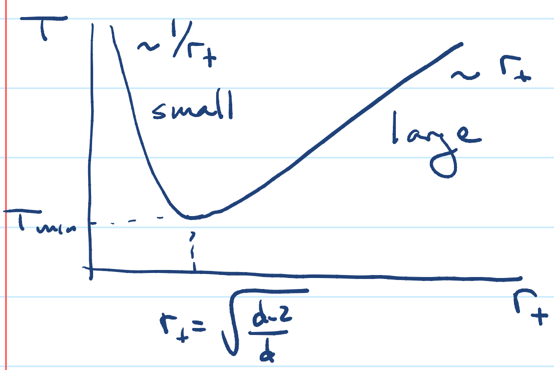
we derived

$$T = \beta^{-1} = \frac{1}{4\pi} f'(r)\Big|_{r=r_H}$$

For the metrics above we find

$$T_H = \frac{1}{4\pi} \frac{r_+^2 d + k(d-2)}{r_+} \overset{L}{\downarrow}= \frac{1}{4\pi} \frac{r_+^2 d + k(d-2)L^2}{r_+ L^2}$$

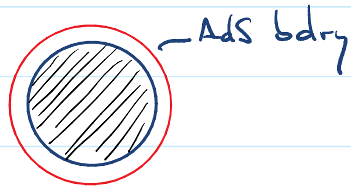
For $k = +1$

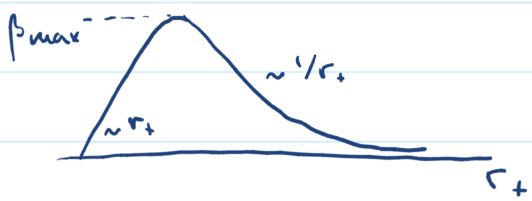


small ~ AF bh

large ~ planar AdS bh

~ black hole in snug box





Curiously, there's a minimum temperature for bhs in AdS, and for $T > T_{min}$ there are two bhs with the same T, which will compete

Thermodynamically.

The minimum temperature is a "box effect":
As the bh grows larger and cools down, it reaches the size of the box and it has to heat up to keep growing.

The two classes of bh have very different properties:

Since we expect that $S = \frac{A_H}{4G} = \frac{1}{4G}\Omega_{d-1} r_+^{d-1}$
(we've proven it in general for E-H gravity)
Then

$$C = T\frac{dS}{dT} = \frac{\Omega_{d-1}}{4\pi} T \frac{d r_+^{d-1}}{dT}$$

and therefore we'll have $\operatorname{sign} C = \operatorname{sign} \frac{dr_+}{dT}$

so

- small AdS bhs have -ve spe heat

  $r_+ \nearrow \; T \searrow$ Thermo unstable

- large AdS bhs have +ve spec heat

  $r_+ \nearrow \; T \nearrow$ Thermo stable (locally)

  Good thermal behavior of dual CFT

  Canonical ensemble well defined

  Large bhs can be in equilibrium w/ radiation and won't evaporate

We can compute the action with counterterm subtraction (see 9903238) subtraction (see 9903258)

[shortcut: compute area and then $S(\beta) = \frac{A_H}{4G}$.

From here one can derive E and F

$$E = \int T\,dS \quad , \quad F = E - TS$$

up to possible additive β-indep integration constants (vacuum energies)]

The result is

$$I = -\frac{\beta\,\Omega_{d-1}}{16\pi G L^2}\, r_+^{d-2}\left(r_+^2 - L^2\right) + \beta E_0$$

→ Casimir

so

$$F - E_0 = -\frac{\Omega_{d-1}}{16\pi G L^2}\, r_+^{d-2}\left(r_+^2 - L^2\right)$$

and we can derive

$$E = \partial_\beta(\beta F) = \frac{d-1}{16\pi G}\,\Omega_{d-1}\,\mu + E_0$$

and

$$S = (\beta\,\partial_\beta - 1)\,\beta F = \frac{\Omega_{d-1}}{4G}\, r_+^{d-1} = \frac{A_H}{4G}$$