The eternal black hole and the thermofield double

jueves, 24 de noviembre de 2022 13:00

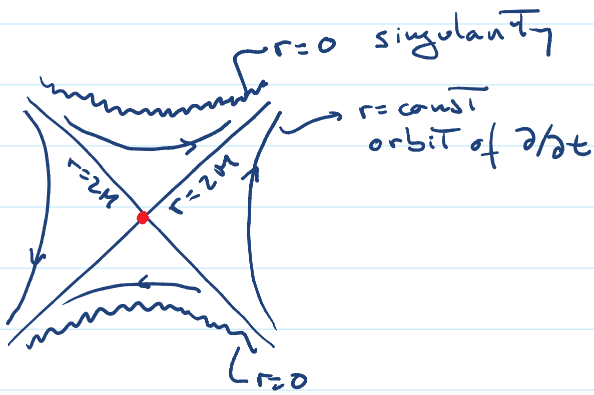
We have argued that AdS black holes describe thermal phases of CFTs. We'll see now that the correspondence actually works at a deeper level, revealing connections that are suggestive of fundamental connections between entanglement and geometry — which we'll elaborate more in later lectures.

Let's first consider what is the boundary geometry of an AdS black hole?

Since we're interested in a state of thermodynamic equilibrium, we'll consider the strictly stationary black hole geometry, with Killing vector $\partial/\partial t$, and maximally analytically extended — the "eternal bh".

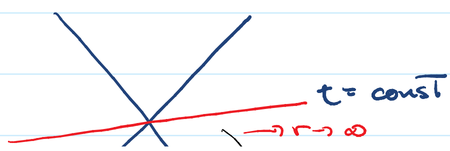
For Schwarzschild, it is known that this is the Kruskal manifold

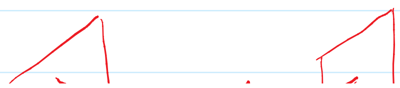
each point in this diagram is a S^2 of radius r

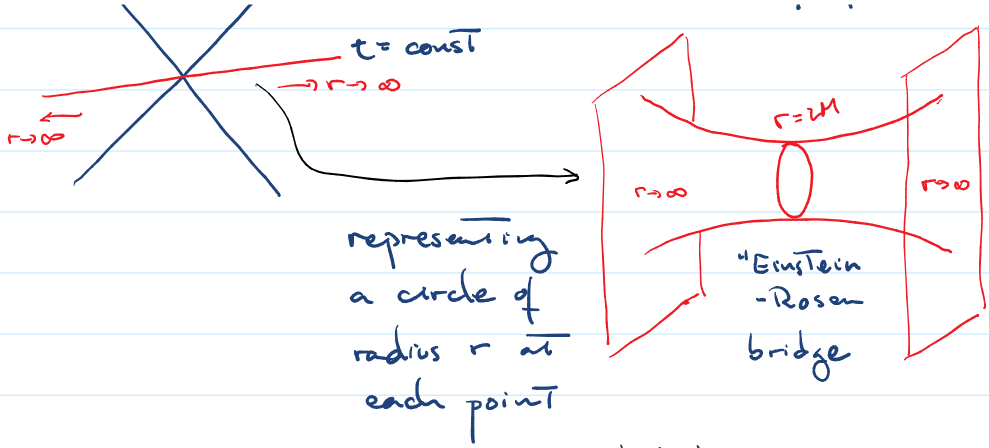


● is the "bifurcation point": a sphere of radius 2M

Constant t spatial sections have two asymptotic regions

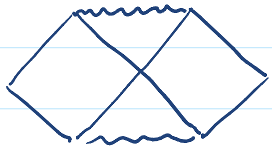






(This is a non-traversable wormhole)

Penrose diagram

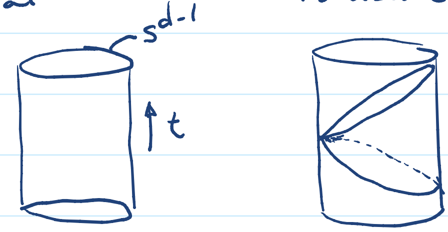


AdS black holes have the same structure near the horizon, but differ in asymptotics.

We saw that AdS has a timelike boundary

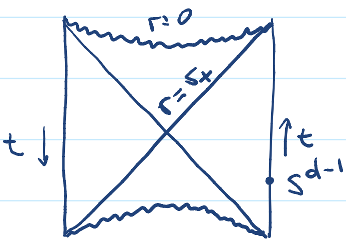
$\mathbb{R}_t \times S^{d-1}$, or $\mathbb{R}_t \times \mathbb{R}^{d-1}$

global Poincaré



AdS black holes have a boundary with two disconnected components, each with geometry

$\mathbb{R}_t \times S^{d-1}$ (or flat)



for global solutions $(k=+1)$ each point in this diagram is S^{d-1}

So we expect that the dual CFT is defined

So we expect that the dual CFT is defined on two disconnected geometries $\mathbb{R}_t \times S^{d-1}$.

That is like having two separate CFTs, identical copies of each other, but on separate spacetimes. Since no signal can be sent from one side to the other, the two CFTs do not interact with each other.

We now want to understand better the state the CFTs are in.

Go back to <u>thermal states in quantum theory.</u>

We describe them as probabilistic states using a density matrix: $e^{-\beta H}$ is an operator (a matrix), which we can normalize

$$\rho = \frac{1}{Z} e^{-\beta H} \qquad Z = \mathrm{Tr}\, e^{-\beta H} = \sum_n \langle n | e^{-\beta H} | n \rangle$$
$$= \sum_n e^{-\beta E_n} \qquad H|n\rangle = E_n |n\rangle$$

In energy eigenbasis the operator is

$$\rho = \frac{1}{Z} \sum e^{-\beta E_n} \, |n\rangle\langle n|$$

$\hookrightarrow$ probability of being in $|n\rangle$

This is not a pure state: it can't be written as $|\psi\rangle\langle\psi|$.

It is a mixed state, with non-zero fine-grained von Neumann entropy $S = -\mathrm{Tr}\, \rho \log \rho$

In quantum theory, mixed states can be "purified".

Consider doubling the Hilbert space $\mathcal{H}$, by tensoring another copy of it $\mathcal{H} \otimes \mathcal{H}$

We have two copies "left" and "right", with a basis of states built as $|n,m\rangle = |n_L\rangle \otimes |m_R\rangle$.

But there exist states that are not direct products of L and R.

Let us consider the "Thermofield double state" in $\mathcal{H}\otimes\mathcal{H}$ built as

$$|TFD\rangle = \frac{1}{\sqrt{Z}} \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R \qquad Z = \sum_n e^{-\beta E_n}$$

(actually we must consider that $|n\rangle_R$ is the CPT reverse of $|n\rangle_L$, but to keep notation simple we'll ignore this)

This is a pure state of $\mathcal{H}\otimes\mathcal{H}$, but it is not a direct product of any $|\psi\rangle_L$ and $|\phi\rangle_R$: it is an entangled state between L and R.

Say that we trace over (ignore) the L side. We get an operator in R:

$$\rho_R = \mathrm{Tr}_L |TFD\rangle\langle TFD| = \sum_{n_L} \langle n_L|TFD\rangle \langle TFD|n_L\rangle$$

$$= \frac{1}{Z} \sum_{\substack{n \\ m \\ p}} \underbrace{\langle n_L|m_L\rangle|m_R\rangle}_{\delta_{nm}|n_R\rangle} e^{-\beta E_m/2} e^{-\beta E_p} \underbrace{\langle P_L|\langle P_R|n_L\rangle}_{\delta_{pn}\ \langle n_R|}$$

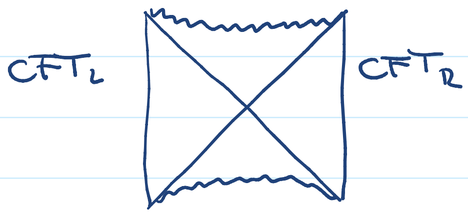
$$= \frac{1}{Z} \sum_n |n\rangle_R {}_R\langle n| e^{-\beta E_n}$$: Thermal density matrix in R

By doubling the system and considering an entangled state, we can describe thermal physics in terms of pure states. The thermal entropy is then seen as entanglement entropy between the two L and R systems.

[In quantum theory all entropy can be regarded as entanglement entropy]

The two systems in the TFD are entangled but do not interact. They cannot exchange signals.

This is very much like what we've found in the eternal AdS black hole



The states of the two CFTs look separately thermal. The boundaries are disconnected, and the theories don't interact.

But their entanglement is represented as the fact that they are connected through the bulk. Quantum entanglement of the two CFTs manifests in a geometrical connection in the bulk. Geometry appears as a consequence of a large amount of entanglement.

The amount of entanglement is quantified by the entropy $S_{BH} = \frac{A_H}{4G} \propto c$ or N^2

In the low-temperature regime, we can also purify the thermal state of the CFT into a TFD. But below the Hawking-Page temperature, the entropy is very small, $S \sim O(N^0) \ll N^2$, and thus there's little entanglement between CFT_L and CFT_R. The dual geometry is two disconnected copies of thermal AdS.